106 年專技高考土木技師考試試題

類科：土木技師

科目：結構設計(包括鋼筋混凝土設計與鋼結構設計)

甲、申論題部分

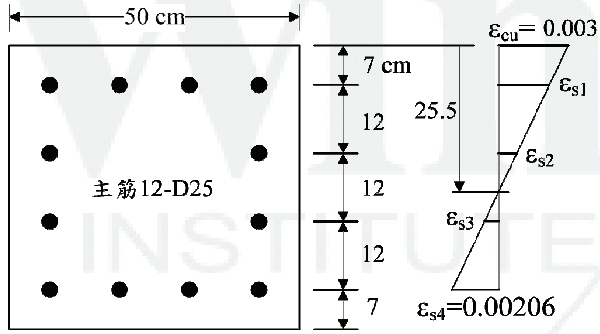
一、已知一鋼筋混凝土方柱斷面寬 50cm 共配置 12 支 D25 主筋，此斷面承受軸壓力 P_a 及彎矩 M_a 同時作用下，斷面應變分布如下圖所示。假設混凝土規定抗壓強度為 $280kgf/cm^2$，主筋 D25 單根鋼筋斷面積為 $5.07cm^2$，鋼筋規定降伏強度為 $4200kgf/cm^2$，鋼筋標稱彈性模數為 $2{,}040{,}000kgf/cm^2$，請回答下列問題：

㈠試問下圖應變狀態之名稱。(5 分)

㈡試計算此應變狀態下各列鋼筋之應變 ε_{s1}、ε_{s2} 及 ε_{s3} 值。(6 分)

㈢試計算此應變狀態下之軸壓力 P_a 及彎矩 M_a 值。(10 分)

㈣試問若柱軸壓力提高為 $2P_a$ 條件下施加彎矩 M_b 發生破壞，請問破壞時中性軸深度是否大於 $25.5cm$？破壞之彎矩 M_b 是否大於 M_a 值？(4 分)



Strain diagram

【擬答】

㈠ $\varepsilon_y = \dfrac{f_y}{E_s} = \dfrac{4200}{2.04\times10^6} = 0.00206 = \varepsilon_{s4}$

即拉力筋正好降伏，此狀態稱為「平衡應變狀態」

㈡ $\varepsilon_{s1} = \dfrac{0.003}{25.5}(25.5-7) = 0.00218$

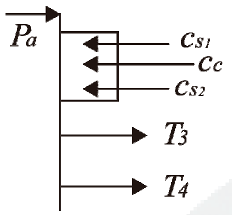
$$\varepsilon_{s2} = \frac{0.003}{25.5}(25.5-19) = 0.00076$$

$$\varepsilon_{s3} = \frac{0.003}{25.5}(31-25.5) = 0.00065$$

㈢ $C_c = 0.85 f_c' b \beta_1 x = 0.85(280)50(0.85)25.5/1000 = 257.9\ tf$

$f_{s1}' = \varepsilon_{s1} E_s = 0.00218(2.04\times10^6) = 4447 > f_y = 4200 kgf/cm^2$

$\therefore f_{s1}' = 4200 kgf/cm^2$



$C_{s1} = (4\times5.07)(f_{s1}' - 0.85 f_c') = 20.28(4200 - 0.85 * 280)/1000 = 80.3\ tf$

$f_{s2}' = \varepsilon_{s2} E_s = 0.0076(2.04\times10^6) = 1550\ kgf/cm^2$

$C_{s2} = (2\times5.07)(f_{s2}' - 0.85 f_c') = 10.14(1550 - 0.85 * 280)/1000 = 13.3\ tf$

$f_{s3} = \varepsilon_{s3} E_s = 0.00065(2.04\times10^6) = 1326\ kgf/cm^2$

$\mathrm{T}_3 = (2\times5.07) f_{s3} = 10.14(1326)/1000 = 13.4\ tf$

$T_4 = (4\times5.07) f_y = 20.28(4200)/1000 = 85.2\ tf$

$\sum F_x = 0$

$\mathrm{P}_a = C_c + C_{s1} + C_{s2} - T_3 - T_4 = 257.9 + 80.3 + 13.3 - 13.4 - 85.2 = 252.9\ tf$

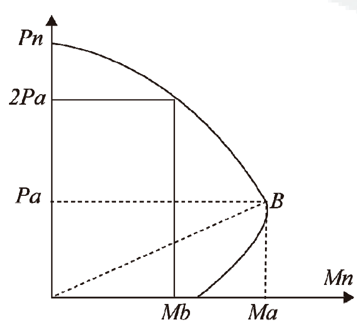
$\sum M_{T4} = 0$

$$\mathrm{P}_a(e + d'') = C_c(d - \frac{1}{2}\beta_1 x) + C_{s1}(d-7) + C_{s2}(d-19) - T_3(12)$$

$$252.9(e+18) = 257.9(43 - \frac{1}{2} * 0.85 * 25.5) + 80.3(43-7) + 13.3(43-19) - 13.4(12)$$

$\therefore e = 26.9\ cm \quad M_a = P_a \cdot e = 252.9(26.9)/100 = 68.0\ tf - m$

㈣依斷面交互影響圖



B 點平衡點 x=25.5cm

當 P_u 提高為 $2P_a$

此時斷面成為壓力

破壞控制

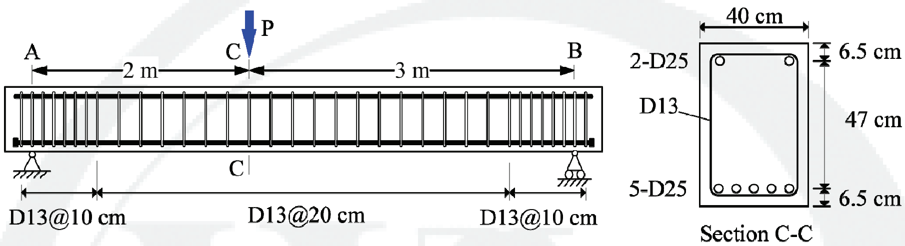
∴ x＞25.5cm

且破壞彎矩 $M_b < M_a$

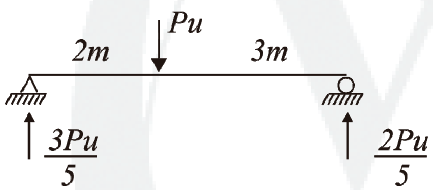
二已知一鋼筋混凝土簡支梁跨度 5m，斷面尺度及配筋如下圖所示。距支承 A 點 2m 處之 C 點承受一集中荷載 P。假設梁自重可忽略，混凝土規定抗壓強度為 $280 kgf/cm^2$，D25 鋼筋單根斷面積為 $5.07 cm^2$，規定降伏強度 f_y 為 $4200 kgf/cm^2$；D13 鋼筋單根斷面積為 $1.27 cm^2$，規定降伏強度 f_{yt} 為 $2800 kgf/cm^2$，請回答下列問題：

㈠若梁底層筋為 5 支 D25 鋼筋，頂層筋為 2 支 D25 鋼筋，試計算斷面 C-C 達到其標稱彎矩 Mn 時之 C 點荷載 P 值。(15 分)

㈡承上，在 C 點荷載 P 值情況下，請評估此梁會不會發生剪力破壞。注意梁兩側箍筋間距為 10cm，中央段箍筋間距為 20cm，均為 D13 閉合箍筋。(10 分)



【擬答】



$$Mu,max = \frac{3}{5} P_u (2m)$$

$$= \frac{6}{5} P_u \quad tf\text{-}m$$

㈠

$$A_s' = 2 \times 5.07 = 10.14 cm^2 \qquad d' = 6.5cm$$

$$A_s = 5 \times 5.07 = 25.35 cm^2 \qquad d = 53.5cm$$

設張力筋降伏，壓力筋未降伏

$$C_c = 0.85 f_c' b \beta_1 x = 0.85(280)40(0.85)x/1000 = 8.09x$$

$$f_s' = \varepsilon_s' E_s = \frac{6120}{x}(x - 6.5)$$

$$C_s = A_s'(f_s' - 0.85 f_c') = 10.14\left[\frac{6120}{x}(x - 6.5) - 0.85(280)\right]/1000 = 59.64 - \frac{403.4}{x}$$

$$T = A_s f_y = 25.35(4200)/1000 = 106.47\ tf$$

$$\sum F_x = 0 \quad C_c + C_s = T \quad \therefore x = 10.53cm$$

Check

$$\varepsilon_s' = \frac{0.003}{10.53}(10.53-6.5) = 0.00115 < \varepsilon_y \quad (OK)$$
$$\varepsilon_s = \frac{0.003}{10.53}(53.5-10.53) = 0.001224 > \varepsilon_y \quad (OK)$$

$$\therefore\ C_c = 8.09(10.53) = 85.19\ tf$$
$$C_s = 59.64 - \frac{403.4}{10.53} = 21.33\ tf$$

$$M_n = C_c(d - \frac{1}{2}\beta_1 x) + C_s(d-d') = 85.19(53.5 - \frac{1}{2}*0.85*10.53) + 21.33(53.5-6.5)$$
$$= 5179\ tf-cm = 51.79\ tf-m$$

$$\because\ \varepsilon_t = \varepsilon_s = 0.01224 > 0.005 \quad \therefore \phi = 0.9$$

$$\phi M_n = 0.9(51.79) = 46.61\ tf-m \ge M_u = \frac{6}{5}P_u \qquad \therefore P_u \le 38.8\ tf$$

(二)

$$\text{A-C 剪力 } \mathrm{V_u} = \frac{3}{5}P_u = \frac{3}{5}(38.8) = 23.3\ tf ----- \mathrm{V}_{u,\max}$$

由中央段間距20cm控制，中央段剪力強度

$$\phi V_c = \phi(0.53\sqrt{f_c'}bd) = 0.75(0.53\sqrt{280})40*53.5/1000 = 14.23\ tf$$

$$\phi V_s = \phi\frac{dA_v f_{yt}}{S} = 0.75\frac{53.5(1.27\times 2)2800}{20}/1000 = 14.27\ tf$$

$$\therefore\ \phi V_n = \phi V_c + \phi V_s = 28.5\ tf > V_{u,\max} = 23.3\ tf$$

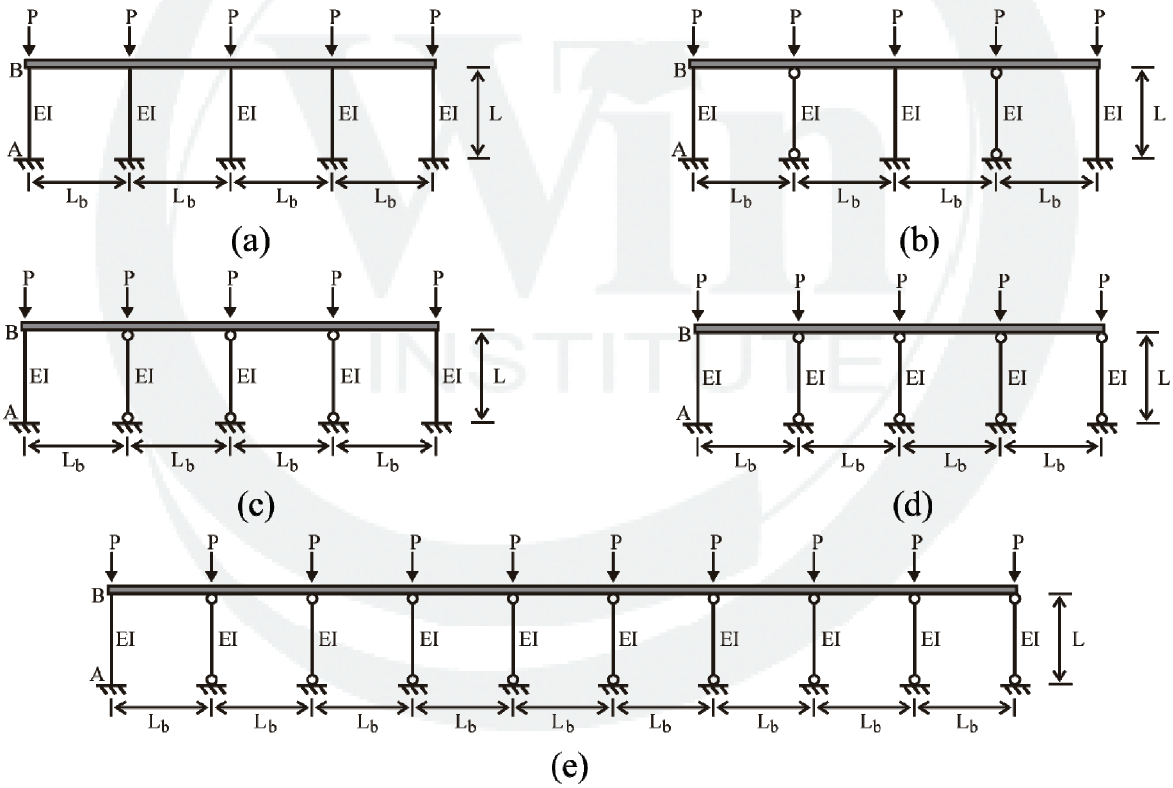
故梁不會發生剪力破壞

三如下圖所示(a)、(b)、(c)、(d)、(e)五個鋼架結構，已知各鋼架結構頂層梁勁度甚大，梁相對柱勁度可視為鋼體；今假設各桿間水平間距L_b夠大，故不需考慮梁之剪力影響。五個鋼架結構中有些柱上下端均為剛接(RigidJoint)，有些則均為鉸接(HingedJoint)，上下端均為鉸接之柱，視為靠桿(LeaningColurmn)；上下端均為剛接柱，視為強柱。

圖(a)中沒有靠柱、圖(b)中有二根靠桿、圖(c)中有三根靠桿、圖(d)中有四根靠桿、圖(e)中有九根靠桿。

請參考策梅厥公式(LeMessurierFormula)，試分別求解圖(a)、圖(b)、圖(c)、圖(d)、圖(e)等各圖中，各鋼架結構中柱 AB 之有效長度係數$K_{(a)}$、$K_{(b)}$、$K_{(c)}$、$K_{(d)}$、$K_{(e)}=?$

今在各圖中柱 AB 之臨界載重$P_{cr}(=\pi^2 EI/(K_{AB}L)^2=P_e/K_{AB}{}^2)$情況下，已知五種鋼架結構之總臨界載重$\sum P_{cr}$可用$P_e$的倍數來表示，試比較總臨界載重$\sum P_{cr(a)}$、$\sum P_{cr(b)}$、$\sum P_{cr(c)}$、$\sum P_{cr(d)}$，說明四者間增加靠桿後具有何意義；另也請比較$\sum P_{cr(d)}$、$\sum P_{cr(e)}$，說明二者間增加靠桿後具有何意義？(25 分)



【擬答】

強柱

$K = 1.0$(上端可移動，不能轉動)

$$P_{er} = \frac{\pi^2 EI}{(1{,}0L)^2} = \frac{\pi^2 EI}{L^2} = P_e$$

靠桿

$K = \infty$

$$P_{er} = \frac{\pi^2 EI}{(\infty L)^2} = o$$

㈠求各構架之 AB 柱的有效長度係數？

圖(a)5 桿均為強柱，荷重均為 P

$\therefore \sum P = 5P$，$\sum P_{ek} = 5(P_e) = 5P_e$

$$K_{(a)} = \sqrt{\frac{P_e}{P_i}\frac{\sum \mathrm{P}}{\sum \mathrm{P}_{ek}}} = \sqrt{\frac{P_e}{P}\frac{5\mathrm{P}}{5\mathrm{P}_e}} = 1$$

圖(b) 有 3 支強柱，2 支靠桿

$\sum P = 5P$，$\sum P_{ek} = 3(P_e) + 2(0) = 3P_e$

$$K_{(b)} = \sqrt{\frac{P_e}{P}\frac{5\mathrm{P}}{3\mathrm{P}_e}} = \sqrt{\frac{5}{3}} = 1.29$$

圖(c) 有 3 支強柱，2 支靠桿

$\sum P = 5P$，$\sum P_{ek} = 2(P_e) + 3(0) = 2P_e$

$$K_{(c)} = \sqrt{\frac{P_e}{P}\frac{5\mathrm{P}}{2\mathrm{P}_e}} = \sqrt{\frac{5}{2}} = 1.58$$

圖(d) 有 1 強柱，4 支靠桿

$\sum P = 5P$，$\sum P_{ek} = 1(P_e) + 4(0) = P_e$

$$K_{(d)} = \sqrt{\frac{P_e}{P}\frac{5\mathrm{P}}{P_e}} = \sqrt{\frac{5}{1}} = 2.23$$

圖(e) 有 1 支強柱，9 支靠桿

$\sum P = 10P$，$\sum P_{ek} = 1(P_e) + 9(0) = P_e$

$$K_{(e)} = \sqrt{\frac{P_e}{P}\frac{10\mathrm{P}}{\mathrm{P}_e}} = \sqrt{\frac{10}{1}} = 3.16$$

㈡求各構架之$\sum P_{cr}$

圖(a) 強柱 $P_{cr}=\dfrac{P_e}{(1)^2}=P_e$ 5支強柱$\sum P_{cr}=5(P_e)=5P_e$

圖(b)強柱$P_{cr}=\dfrac{P_e}{(1.29)^2}=0.6P_e$靠桿$P_{cr}=0$

3 支強柱+2 支靠桿$\sum P_{cr}=3(0.6P_e)+2(0)=1.8P_e$

圖(c) 強柱$P_{cr}=\dfrac{P_e}{(1.58)^2}=0.4P_e$

2 支強柱+3 支靠桿$\sum P_{cr}=2(0.4P_e)+3(0)=0.8P_e$

圖(d) 強柱$P_{cr}=\dfrac{P_e}{(2.23)^2}=0.2P_e$

1 支強柱+4 支靠桿$\sum P_{cr}=0.2P_e+4(0)=0.2P_e$

圖(e) 強柱$P_{cr}=\dfrac{P_e}{(3.16)^2}=0.1P_e$

1 支強柱+9 支靠桿$\sum P_{cr}=0.1P_e+9(0)=0.1P_e$

註：本題非屬結構設計範圍

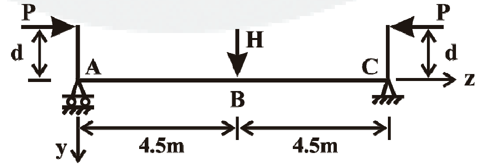
四、已知一型鋼梁柱構材，相關尺寸配置如下圖所示，假設 A、B、C 三點均有側向支撐補強，且此型鋼梁柱構材成受偏心 d 之工作載重壓力 P 及橫向工作載重 H。若此梁柱構材採用 W14×30 結實斷面、$H=2tf$ 、$P=15tf$ 、$d=25cm$ ，試以容許應力法(ASD)，驗核此型鋼梁柱構材強度是否滿足規範？(25 分)

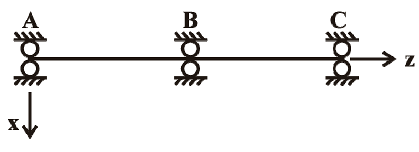
W14×30之斷面及材料性質：

$A=57.1cm^2$，$d=35.2cm$，$t_w=0.686cm$，$b_f=17.1cm$，$t_f=0.98cm$，$r_t=4.45cm$

$I_x=12100cm^4$，$S_x=688cm^3$，$r_x=14.6cm$，$I_y=816cm^4$，$S_y=95.4cm^3$，$r_y=3.78cm$

$E=2040\,tf/cm^2$，$F_y=2.5\,tf/cm^2$，$F_u=4.1\,tf/cm^2$

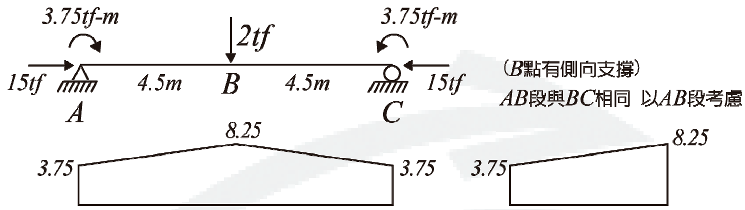




【擬答】

$$P_d = 15(25)/100 = 3.75\,tf-m = M_A = M_c$$

$$H = 2\,tf$$



$$M_B = 3.75 + \frac{1}{4}(2)9 = 8.25\,tf-m$$

$$f_a = \frac{P}{A} = \frac{15}{57.1} = 0.26\,tf/cm^2$$

F_a：

$$(\frac{KL}{\gamma})_x = \frac{1.0(900)}{14.6} = 61.6 \quad (\frac{KL}{\gamma})_y = \frac{1.0(450)}{3.78} = 119.0 \leftarrow$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} = \sqrt{\frac{2\pi^2(2040)}{2.5}} = 126.9 > 119.0$$

$$F_a = \frac{\left[1 - \frac{1}{2}(\frac{\frac{KL}{\gamma}}{C_c})^2\right]F_y}{\frac{5}{3} + \frac{3}{8}(\frac{\frac{KL}{\gamma}}{C_c}) - \frac{1}{8}(\frac{\frac{KL}{\gamma}}{C_c})^3} = 0.73\ tf/cm^2$$

$$\therefore \frac{f_a}{F_a} = \frac{0.26}{0.73} = 0.356 > 0.15$$ 須考慮二次力矩

$$f_b = \frac{M}{S_x} = \frac{8.25(100)}{688} = 1.2\,tf/cm^2$$

$$L_c = \begin{cases} \frac{20bf}{\sqrt{F_y}} = \frac{20(17.1)}{\sqrt{2.5}} = 216cm \quad \leftarrow \\ \frac{1400}{d/A_f\ F_y} = \frac{1400}{35.2/(17.1\times 0.98)*2.5} = 267cm \end{cases}$$

$\therefore L_b = 450cm > L_c$ 無充分側向支撐

$$C_b = 1.75 + 1.05(\frac{M_1}{M_2}) + 0.3(\frac{M_1}{M_2})^2 = 1.75 + 1.05(\frac{-3.75}{8.25}) + 0.3(\frac{-3.75}{8.25})^2 = 1.33$$

$$\frac{L_b}{\gamma_T}=\frac{450}{4.45}=101.1$$

$$\sqrt{\frac{7160C_b}{F_y}}=6.17 \qquad \sqrt{\frac{35800C_b}{F_y}}=138.0 \therefore \sqrt{\frac{7160\,C_b}{F_y}}<\frac{L_b}{\gamma_T}<\sqrt{\frac{35800\,C_b}{F_y}}$$

$$F_b=\begin{cases}\left[\frac{2}{3}-\frac{F_y(L_b/\gamma_T)^2}{107600\,C_b}\right]F_y=1.22\,tf/cm^2 \leftarrow \text{取大者}\\ \frac{840\,C_b}{L_b\,d/A_f}=\frac{840(1.33)}{450[35.2/(17.1\times 0.98)]}=1.18\,tf/cm^2\end{cases}$$

但端點視為有充分側向支撐

$$\frac{b_f}{2tf}=\frac{17.1}{2(0.98)}=8.7<\frac{17}{\sqrt{F_y}}=10.8\ \ (OK)$$

$$\frac{d}{tw}=\frac{35.2}{0.686}=51.3<\frac{170}{\sqrt{F_y}}(1-3.74\frac{f_a}{F_a})=65.7\quad (OK)$$

為結實斷面 $F_b=0.66F_y=1.65\,tf/cm^2$

$$C_m=0.6-0.4(\frac{M_1}{M_2})=0.6-0.4(\frac{-3.75}{8.25})=0.78$$

$$F_e'=\frac{12\pi^2E}{23(KL/\gamma)_x{}^2}=\frac{12\pi^2(2040)}{23(61.6)^2}=2.77\,tf/cm^2$$

$$\frac{C_m}{1-f_a/F_e'}=\frac{0.78}{1-(0.26/2.77)}=0.86$$

Check：

$$\frac{f_a}{F_a}+\frac{C_m}{1-f_a/F_e'}(\frac{f_b}{F_b})=0.356+0.86(\frac{1.2}{1.22})=1.2>1.0\quad (N.G)$$

端點：

$$\frac{f_a}{0.6F_y}+\frac{f_b}{F_b}=\frac{0.26}{0.6(2.5)}+\frac{1.2}{1.65}=0.9<1.0$$

$\boxed{\text{結論}}$：$W14\times 30$ 斷面無法滿足要求

※參考公式：請自行選擇適合使用的公式，若有問題請自行修正。

$$P_{ek}=\frac{\pi^2EI}{(kL)^2}=\frac{p_e}{(k)^2}\ ;\ B_2=\frac{1}{1-\sum p_u/\sum p_{ek}}\ ;$$

萊梅厥公式(LeMessurierformula)：$K_{i,\text{modified}}=\sqrt{\frac{P_e}{P_i}\frac{\sum P}{\sum P_{ek}}}$

$r_T=\sqrt{\frac{I_y/2}{A_f+A_w/6}}$，$A_w=(d-2t_f)t_w$

$$\frac{20b_f}{\sqrt{F_y}}\ or\ \frac{1400}{(d/A_f)F_y}\ ；C_b = 1.75 + 1.05(M_A/M_B) + 0.3(M_A/M_B)^2 \le 2.3$$

$$\sqrt{7160C_b/F_y} \le l/r_t \le \sqrt{35800C_b/F_y}\ ，then\ F_b = \left[2/3 - \frac{F_y(l/r_t)^2}{107600C_b}\right]F_y \le 0.6F_y$$

$$\sqrt{35800C_b/F_y} < l/r_t\ ，than\ F_b = 12000C_b/(l/r_t)^2 \le 0.6F_y$$

$$F_b = \frac{840C_b}{(Ld/A_f)}\ ；C_m = 0.6 - 0.4(M_A/M_B)(\ge 0.4)$$

(1) $\dfrac{f_a}{F_a} \le 0.15$，$\dfrac{f_a}{F_a} + \dfrac{f_{bx}}{F_{bx}} + \dfrac{f_{by}}{F_{by}} \le 1$

(2) $\dfrac{f_a}{F_a} > 0.15$，$\dfrac{f_a}{F_a} + \dfrac{C_{mx}f_{bx}}{(1-\frac{f_a}{F'_{ex}})F_{bx}} + \dfrac{C_{my}f_{by}}{(1-\frac{f_a}{F'_{ey}})F_{by}} \le 1$，$F'_e = \dfrac{12\pi^2E}{23(KL_b/r_b)^2}$

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1$$

$$For\ \frac{KL}{r} \le C_c\ ；F_a = \frac{\left[1 - \frac{(KL/r)^2}{2C_c^{\,2}}\right]F_y}{\frac{5}{3} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c^{\,3}}}\ ；For\ \frac{KL}{r} > C_c\ \ F_a = \frac{12\pi^2E}{23(KL/r)^2}$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}}\ ；For\ \lambda_c \le 1.5，F_{cr} = (0.658^{\lambda_c^{\,2}})F_y\ ；\mathrm{For}\ \lambda_c > 1.5，F_{cr} = \frac{0.877}{\lambda_c^{\,2}}F_y$$

$$L_p = \frac{80r_y}{\sqrt{F_y}}\ ，L_r = \frac{r_yX_1}{(F_y - F_r)}\sqrt{1 + \sqrt{1 + X_2(F_y - F_r)^2}}$$

$$M_n = C_b\left[M_p - (M_p - M_r)(\frac{L_b - L_p}{L_r - L_p})\right] \le M_p\ ，M_n = \frac{C_bS_xX_1\sqrt{2}}{L_b/r_y}\sqrt{1 + \frac{X_1^{\,2}X_2}{2(L_b/r_y)^2}} \le M_p$$

$$M_r = (F_y - F_r)S_x\ ；X_1 = \frac{\pi}{S_x}\sqrt{EGJA/2}\ ，X_2 = 4\frac{C_w}{I_y}\left[S_x/(GJ)\right]^2$$

$For\ flange$：$\lambda_{pd} = 14/\sqrt{F_y}$，$\lambda_p = 17/\sqrt{F_y}$，$\lambda_r = 25/\sqrt{F_y}$ (F_y單位$= tf/cm^2$)

$For\ web$：$\lambda_p = \dfrac{170}{\sqrt{F_y}}(1 - 3.74\dfrac{f_a}{F_y})$ $for\ f_a/F_y \le 0.16$；$68/\sqrt{F_y}$ $for\ f_a/F_y > 0.16$

祝金榜題名