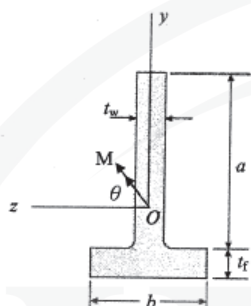


108 年專技高考土木技師試題

類科：土木技師

科目：結構分析(包括材料力學與結構學)

一、倒 T 型斷面如下圖所示。 $b = 200\text{mm}$ ， $a = 300\text{mm}$ ， $t_w = 20\text{mm}$ ， $t_f = 25\text{mm}$ 。受彎矩 $M = 20\text{kN} \cdot \text{m}$ ，角度 $\theta = 45^\circ$ 。請計算該斷面最大之拉應力與壓應力。(25 分)



【擬答】

$$Y_b = \frac{200 \times 25 \times 12.5 + 300 \times 20 \times (150 + 25)}{200 \times 25 + 300 \times 20} = 101.14\text{mm}$$

$$Y_t = 325 - 101.14 = 223.86\text{mm}$$

$$M_z = 20 \cos 45^\circ = 14.142\text{kN} \cdot \text{m}$$

$$M_y = 20 \sin 45^\circ = 14.142\text{kN} \cdot \text{m}$$

$$I_z = \frac{1}{12} \times 200 \times 25^3 + 200 \times 25 \times (Y_b - 12.5)^2 + \frac{1}{12} \times 20 \times 300^3 + 20 \times 300 \times (Y_t - 150)^2$$

$$= 117,277,462\text{mm}^4$$

$$I_y = \frac{1}{12} \times 25 \times 200^3 + \frac{1}{12} \times 300 \times 20^3 = 16,866,667\text{mm}^4$$

斷面 4 個最外緣點位之應力：

$$\text{左下角 A 點 } \sigma_A = \frac{M_z \times 101.14}{I_z} + \frac{M_y \times 100}{I_y} = 12.196 + 83.85 = 96.05\text{MPa}$$

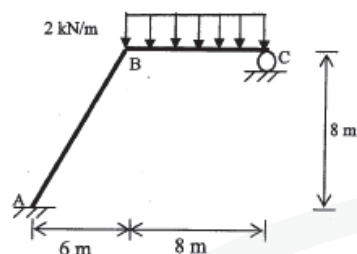
$$\text{右下角 C 點 } \sigma_C = \frac{M_z \times 101.14}{I_z} - \frac{M_y \times 100}{I_y} = 12.196 - 83.85 = -71.65\text{MPa}$$

$$\text{左上角 D 點 } \sigma_D = \frac{-M_z \times 223.86}{I_z} + \frac{M_y \times 10}{I_y} = -26.99 + 8.385 = -18.6\text{MPa}$$

最大壓應力為 71.65 MPa

Diagram of a continuous beam with six supports (A, B, C, D, E, F) and various loads. The beam is divided into segments of 10m, 5m, 5m, 10m, 10m, 10m, and 10m. The diagram shows the distribution of reactions (R_A, R_D, R_E, R_F), shear force (S_{BC}, S_{BG}), and bending moment (M_B) along the beam. The reactions are given as $R_A=1, R_D=1, R_E=1, R_F=1$. The shear force is given as $S_{BC}=0, S_{BG}=0$. The bending moment is given as $M_B=0, S_{BC}=\frac{M_B}{5}, S_{BG}=0$. The diagram also shows the distribution of shear force and bending moment along the beam segments.

三、請利用力法(諧合變位法)計算 C 點支承反力，並繪製桿件 BC 的彎矩圖。下圖各桿件之 EI 均相同。(25 分)



【擬答】

設 C 點反力 R_C 為質力

分段	原點	範圍	M'	m
AB	B	0~10	$-64 + \frac{48}{5}x$	$8 + \frac{3}{5}x$
BC	C	0~8	$-x^2$	x

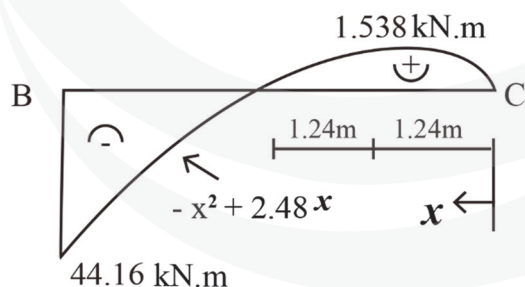
$$\Delta_{cy} = \frac{1}{EI} \left[\int M' m dx + \left(\int m^2 dx \right) \times R_C \right] = 0$$

$$\Rightarrow \left[\int_0^{10} \left(-64 + \frac{48}{5}x \right) \left(8 + \frac{3}{5}x \right) dx + \int_0^8 (-x^2) x dx \right] + \left[\int_0^{10} \left(8 + \frac{3}{5}x \right)^2 dx + \int_0^8 x^2 dx \right] \times R_C = 0$$

解得 $R_C = 2.48 kN$

AB 段彎矩，以 B 為原點 ($0 \leq x \leq 10$)

$$M_{AB} = \left(-64 + \frac{48}{5}x \right) + \left(8 + \frac{3}{5}x \right) \times 2.48 = -44.16 + 11.088x$$

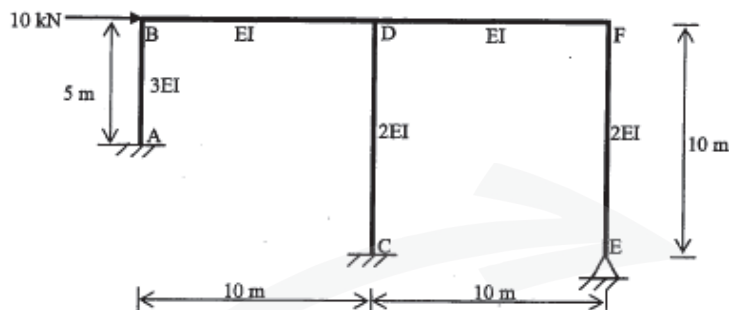


BC 段彎矩，以 C 為原點 ($0 \leq x \leq 8$)

$$M_{BC} = -x^2 + 2.48x$$

答：C 點支承反力為 $2.48 kN(\uparrow)$

四請利用力矩分配法計算各桿件端點力矩。桿件 EI 值比例標示於下圖。(25 分)



【擬答】

設側位移為 Δ

側移的固定端力矩

$$\begin{aligned} M_{AB}^F : M_{CD}^F : M_{EF}^F &= -\frac{6(3EI)\Delta}{5^2} : -\frac{6(2EI)\Delta}{10^2} : -\frac{3(2EI)\Delta}{10^2} \\ &= -\frac{72EI\Delta}{100} : -\frac{12EI\Delta}{100} : -\frac{6EI\Delta}{100} = -12a : -2a : -a \\ a &= \frac{6EI\Delta}{100} \end{aligned}$$

桿件勁度比：

$$\begin{aligned} S_{AB} : S_{BD} : S_{CD} : S_{DF} : S_{EF} \\ &= \frac{4 \times 3EI}{5} : \frac{4 \times EI}{10} : \frac{4 \times 2EI}{10} : \frac{4 \times EI}{10} : \frac{3 \times 2EI}{10} = 24 : 4 : 8 : 4 : 6 \\ &= 12 : 2 : 4 : 2 : 3 \end{aligned}$$

結點	A	B		D			C	F	
EM	AB	BA	BD	DB	DF	DC	CD	FD	FE
D.F		6/7	1/7	1/4	1/4	1/2	0	2/5	3/5
F.E.M	-12	-12				-2	-2		-1
D.M		10.288	1.714	0.5	0.5	1		0.4	0.6
C.O.M	5.144		0.25	0.857	0.2		0.5	0.25	
D.M		0.214	-0.036	-0.264	-0.264	-0.529		-0.1	-0.15
C.O.M	-0.107		-0.132	-0.018	-0.05		-0.264	-0.132	
D.M		0.113	0.019	0.017	0.017	0.034		0.0528	0.0792
C.O.M	0.0565		0.0085	0.0095	0.0264		0.017	0.0085	
D.M.		-0.0173	-0.0012	-0.109	-0.009	-0.018		-0.0034	0.0051
$\Sigma M'$	-6.9065	-1.8203	+1.8223	+1.0925	+0.04204	-1.513	-1.747	+0.4759	-0.4759

$$\begin{aligned}
 \text{由力平衡：} & \frac{M_{BA} + M_{AB}}{5} + \frac{M_{CD} + M_{DC}}{10} + \frac{M_{FE}}{10} + 10 = 0 \\
 \Rightarrow & \left(\frac{-6.9065 - 1.8203}{5} + \frac{-1.513 - 1.747}{10} + \frac{-0.4759}{10} \right) \times a + 10 = 0 \\
 \Rightarrow & a = \frac{10}{2.11895} = 4.7193
 \end{aligned}$$

a 代回表格中得各桿端點力矩：

$$M_{AB} = -32.6 \text{ kN.m}$$

$$M_{BA} = -8.59 \text{ kN.m}$$

$$M_{BD} = +8.59 \text{ kN.m}$$

$$M_{DB} = 5.16 \text{ kN.m}$$

$$M_{DF} = 1.98 \text{ kN.m}$$

$$M_{DC} = -7.14 \text{ kN.m}$$

$$M_{CD} = -8.24 \text{ kN.m}$$

$$M_{FD} = +2.25 \text{ kN.m}$$

$$M_{FE} = +2.25 \text{ kN.m} \circ$$

祝金榜題名 最新名師線上解題&開課

